Intrinsic Low Thermal Conductivity and Phonon Renormalization Due to Strong Anharmonicity of Single-Crystal Tin Selenide

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ABSTRACT: Two-dimensional (2D) van der Waals material tin selenide (SnSe) has recently attracted intensive interest due to its exceptional thermoelectric performance. However, the thermal properties and phonon transport mechanisms in its single-crystal form remain elusive. Here, we measured high-quality SnSe single crystals using nanoscale thermometry based on ultrafast optical spectroscopy and found that its intrinsic thermal conductivity is highly anisotropic in different crystallographic directions. To quantify phonon anharmonicity, we developed a new experimental approach combining picosecond ultrasonics and X-ray diffraction to enable direct measurement of temperature-dependent sound velocity, thermal expansion coefficient, and Grüneisen parameter. The measured Grüneisen parameter suggests an abnormally large temperature effect on phonon dispersion that contributes to over 90% of phonon frequency shifts. Furthermore, we performed ab initio calculations using different methods: in comparison with self-consistent phonon theory, the harmonic and quasi-harmonic models that have been widely used in current phonon calculations fail to accurately predict these important thermophysical properties at room temperature and below. Our study reveals an extremely strong intrinsic anharmonicity in SnSe that introduces phonon renormalization near room temperature. This study represents an important research benchmark in characterizing high-performance thermal energy materials and provides fundamental insight into advancing modern calculation methods for phonon transport theory.

KEYWORDS: thermal transport, phonon softening, ab initio theory and quasi-harmonic approximation, thermoelectric, two-dimensional materials, picosecond acoustic microscopy

Layered van der Waals materials have been intensively investigated for their exotic properties.1 Very recently, tin selenide (SnSe) has drawn a lot of attention because of the reported exceptional thermoelectric properties.2–24 For thermoelectrics, the heat-to-electricity energy conversion efficiency of SnSe is improved by a very low thermal conductivity. However, the fundamental mechanisms behind the ultralow thermal conductivity in SnSe have remained elusive. Experimentally, different results have been reported in the literature due to varied crystal quality including mass density, polycrystallinity, doping level, carrier density, defects, phase mixing, etc.2,5–14,18–24 Theoretically, in semiconductors, thermal transport is generally considered as the entropy propagation and interactions of phonons, i.e., the quantum-mechanical modes of lattice vibrations.25–29 Under current phonon theory, most calculations take harmonic models2,12,13,30 that are limited to quadratic terms in the interatomic potential or quasi-harmonic models15–17 that consider volume-dependent phonon frequencies to account for thermal expansion but does not take account of the phonon frequency renormalization due to phonon-phonon interactions caused by high-order anharmonicities. Therefore, it remains unclear if these theory calculations can be sufficient to explain the behaviors in those materials with strong anharmonicity like SnSe. To clarify the confusion and understand the intrinsic thermal property, here we perform nanoscale anisotropic thermal transport measurement and ab initio calculation on undoped and fully dense single crystals of SnSe.

To investigate the intrinsic properties, it is key to achieving high-quality crystals. A multistep recrystallization process was used to minimize defects and improve crystalline density (see Supporting Information). Figure 1a shows an optical image of as-grown SnSe samples, and Figure 1b depicts its crystal structure. At room temperature, single-crystal SnSe is a layered...
material with an orthorhombic structure, which is classified as \textit{Pnma} space group, \#62. X-ray diffraction (XRD) measurement on the \(a\)-axis cleavage plane (Figure 1c) clearly shows the diffraction peaks from lattice planes (400), (600), and (800), indicating that the surface plane is perpendicular to the \(a\)-axis and that no defects and grains are observable. In addition, we measured the mass density of the as-grown SnSe sample using Archimedes' principle. The measured value, \(6.16 \pm 0.05 \text{ g/cm}^3\).
conductivity was only observed in some amorphous phases. To better understand the transport mechanism for phosphorus, the cross-plane thermal conductivity of SnSe is about one-order smaller. It is also about three times smaller than the in-plane and the histogram distribution is summarized in Figure 2c. The good agreement between the experimental and guideline data clearly indicates that the defect and boundary scattering are negligible, which verifies the high quality single-crystal nature of the SnSe samples used. Therefore, in these high-quality SnSe single crystals, anharmonic phonon scattering dominates thermal transport, and the extremely low thermal conductivity indicates strong anharmonicity.

To further quantitatively investigate the strong anharmonicity of SnSe, we start to look at the special features and unique physics of high anharmonicity materials from a theoretical point of view. Fundamentally, the phonon picture was following Albert Einstein’s seminal idea of using quantized harmonic oscillator to describe atomic vibrations in solids: Phonons are defined as quantized normal modes resulting from harmonic forces between atoms. In most literature, harmonic models that are limited to quadratic terms in the interatomic potential are used to derive phonon frequencies. In this case, the phonon dispersion relation is invariant to temperature. This holds good approximation for most materials but may not be valid for materials with strong anharmonicity. With non-negligible anharmonic forces, the phonon dispersion will change with temperature, which results in change of sound velocity. To quantify the anharmonicity, we measured the sound velocity at different temperature and extract the mode-dependent Grüneisen parameter at the Brillouin zone center (Γ point) from the temperature-dependent sound velocity.

The mode-dependent Grüneisen parameter \( \gamma_{\lambda} \) is a key metric to quantify the anharmonicity, which characterizes the relationship between phonon frequency and volume change:

\[
\gamma_{\lambda} = -\frac{\partial \ln \omega_{\lambda}}{\partial V}
\]  

where \( \lambda = (\mathbf{q}, j) \) labels a phonon mode with wave vector \( \mathbf{q} \) and polarization \( j \). \( \omega_{\lambda} \) is the frequency of the mode \( \lambda \), and \( V \) is the

\[
\kappa = \frac{1}{3}C_v\tau
\] 

where \( C_v \), \( \nu \), and \( \tau \) are the specific heat, sound velocity, and phonon relaxation time, respectively. The total phonon relaxation time (\( \tau_{\nu} \)) is contributed by Rayleigh scattering (\( \tau_r \)) from impurities and defects, anharmonic (Umklapp) phonon scattering (\( \tau_0 \)), and grain boundary scattering (\( \tau_b \)), following the Matthiessen rule and Callaway model:

\[
\tau_{\nu}^{-1} = \tau_r^{-1} + \tau_0^{-1} + \tau_b^{-1} = A\omega^4 + B\omega^2 T + \frac{\nu}{L}
\]
volume of the unit cell. Recent literature calculations have reported widely varying values for the Grüneisen parameter—from 0.5 to 6.5.3,12 Generally, a small Grüneisen parameter value (γ ≈ 1) at the Brillouin zone center (Γ point: ω_β ≈ 0) is expected for most normal materials.38 At the Brillouin zone center (Γ), phonon dispersion is linear (ω_j = v_j q) where the phonon group velocity equals to the speed of sound (v_j). Therefore, the mode-dependent Grüneisen parameter at Γ point is (see Supporting Information)

\[ \gamma_j = \frac{1}{\alpha_j} \left( \frac{\Delta v_j}{v_j} \right) \]

where \( \alpha_j \) and \( v_j \) are the volumetric thermal expansion coefficient, linear expansion coefficient, and sound velocity, respectively. In Equation 4, \( \gamma_j \) contains the thermal expansion induced phonon frequency shifts (i.e., the quasi-harmonic contribution), and a second term of anharmonicity induced frequency shifts. The difference of those two contributions will be further discussed later. Equation 4 enabled us to measure the temperature-dependent Γ point Grüneisen parameter \( \gamma_\Gamma \) for the first time of acoustic phonons.

Here, we developed an experimental approach to directly measure the mode-dependent Grüneisen parameter of the Brillouin zone center at different temperature using picosecond ultrasonics and XRD. Note that most literature are limited to measuring the macroscopic Grüneisen parameter (i.e., weighted average of mode-dependent \( \gamma_j \) by heat capacity contributions), or probing Raman shifts for optical branch phonons. To evaluate the temperature effect of the ultralow cross-plane thermal conductivity in single-crystal SnSe, we focused on the longitudinal phonon branch along the a-axis and measured the sound velocity, lattice thermal expansion, and specific heat from 120 to 300 K. First, the sound velocity was measured using picosecond laser ultrasonic (PLU) measurement. Figure 3a illustrates the working principle of PLU, where a laser pump generates a picosecond acoustic pulse from the surface that penetrates into the sample; the acoustic wave reaches the interface between SnSe and the substrate and its partial reflection creates sound echoes. The sound echoes return to the surface and are detected by a laser probe (Figure 3b). The time delay (\( \Delta t_{echo} \)) between echoes is measured and contributed to by the round trip of the acoustic wave in the sample film with thickness \( d \). Therefore, the sound velocity is calculated by

\[ v_s = 2d/\Delta t_{echo} \]

For the PLU study, exfoliated SnSe films were transferred on a SiO_2 substrate for measurement, and sample thickness was measured by atomic force microscopy (AFM) (Figure 3c). The evolution of the PLU signal between temperatures of 120 to 300 K was plotted as a 2D map (Figure 3d) and clearly shows that \( \Delta t_{echo} \) is reduced at low temperature. The sound velocity was measured from 3343 m/s at 300 K to 3422 m/s at 120 K (Figure 3e). These measurements show a significant 2.4% change in temperature-dependent sound velocity (thus, the phonon dispersion relation) of SnSe in comparison with other common materials (for example, 0.46% for silicon). In addition, the measured sound velocity of SnSe is actually over 50% larger than that predicted from the density functional theory (DFT) (2117 m/s²). Next, we directly measured the volumetric and linear thermal expansion coefficients along each crystal orientation of the SnSe lattice, using temperature-dependent XRD (Figure 3f). Finally, using eq 4, we extracted the temperature-dependent Γ point Grüneisen parameter (\( \gamma_\Gamma \)) (Figure 3g).

From the measurement (Figure 3g), a strong temperature effect was observed for \( \gamma_\Gamma \). The measured \( \gamma_\Gamma \) is 4.31 at room temperature and increases to 4.97 at 120 K, which indicates a very strong phonon anharmonicity. Such an experimental

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**Figure 3.** Temperature-dependent measurements of sound velocity, thermal expansion, and Grüneisen parameter of single-crystal SnSe. (a) Schematic of picosecond laser ultrasonic measurement. (b) Picosecond laser ultrasonic data along the a-axis. The interval (\( \Delta t_{echo} \)) between the peak and valley echoes is the time for a round trip of the acoustic wave inside the SnSe sample. The acoustic wave (longitudinal) is generated at the top surface and partially reflected at the SnSe and substrate interface. (c) Atomic force microscopy image of a thin film SnSe sample, measuring the sample thickness of 884 nm. (d) Two-dimensional mapping of the temperature-dependent ultrasonic data along a-axis of SnSe. (e) Temperature-dependent longitudinal sound velocity (a-axis) of SnSe. (f) Linear and volumetric thermal expansion coefficients in each direction. (g) Temperature-dependent Grüneisen parameter (\( \gamma_\Gamma \)) of SnSe.
Figure 4. Ab initio calculations of temperature-dependent phonon band structures of SnSe using harmonic approximation (HA), quasi-harmonic approximation (QHA), and anharmonic approximation (AH). (a) Schematic of interatomic potential as a function of distance, illustrating HA and QHA. (b) Phonon dispersion relation of SnSe at 300 K from density functional theory (DFT) in comparison with experimental data from inelastic neutron scattering (INS). (c–e) Temperature-dependent phonon band structures using different models: (c) AH only considering the anharmonicity, (d) QHA only considering the thermal expansion, and (e) AH + QHA model considering both anharmonicity and thermal expansion. (f) Ab initio predictions of temperature-dependent sound velocity and thermal expansion coefficient of SnSe from different models, in comparison with experimental data. For illustration clarity, the sound velocity is normalized at 120 K; HA result is independent with temperature; while AH shows a strong temperature dependence comparable with experiment.

The observation has important insights to the state-of-the-art phonon theory calculations. Mathematically, the interatomic potential \( U \) of a crystal can be expanded with respect to atomic displacements from the equilibrium positions of atoms

\[
U = U_0 + \frac{1}{2} \frac{\partial^2 U}{\partial r_i^a \partial r_j^b} u_i^a u_j^b + \frac{1}{3!} \frac{\partial^3 U}{\partial r_i^a \partial r_j^b \partial r_k^c} u_i^a u_j^b u_k^c + \ldots
\]

where \( U_0 \) is the interatomic potential at the equilibrium, \( r_i^a \) denotes the coordinate of the \( i \)-th nucleus along Cartesian direction \( a \), and \( u_i^a \) denotes the atomic displacement from equilibrium position. The \( n \)-th order derivative of the interatomic potential with respect to displacement is the interatomic force constants (IFCs).

Most current theoretical methods generally use the harmonic (HA) model

\[
D(q)\epsilon_q = \omega_q^2 \epsilon_q
\]

where \( \epsilon_q \) and \( \omega_q \) are the phonon eigenvector and phonon frequency corresponding to the phonon wave vector \( q \). \( D(q) \) is the dynamical matrix determined by second-order IFCs and atomic masses. HA only considers the parabolic expansion of interatomic potential, where forces acting on each atom linearly respond to the atomic displacements (Figure 4a). Combined with the DFT calculated second-order IFCs, HA can precisely predict the phonon dispersion for 0 K but neglects the temperature effect. However, the phonon frequency will shift with the increase of temperature. The decrease of phonon frequency with increasing temperature is called phonon softening and the opposite is called phonon hardening. In general, phonon frequency shift is contributed by two mechanisms: the thermal expansion and the intrinsic anharmonicity.

Thermal expansion stems from the thermal kinetic energy of the atoms in the crystal. With the rising of temperature, the interatomic distance usually expands due to the increased amplitude of the lattice vibrations, thereby shifting the phonon frequency. Another popular approach, the quasi-harmonic model (QHA), assumes harmonic oscillators and covers the frequency shift due to thermal expansion

\[
\Delta \omega_{\text{QHA}}^0 \approx -\omega_0^0 \int_0^T \alpha_v(T) \, dT
\]

where \( \lambda = (q, j) \) labels a phonon mode with wave vector \( q \) and polarization \( j \). \( \omega_0^0 \) and \( \Delta \omega_{\text{QHA}}^0 \) are the 0 K phonon frequency and quasi-harmonic frequency shift of phonon mode \( \lambda \), respectively. \( \gamma_\lambda \) is the Grüneisen parameter of phonon mode \( \lambda \). \( \alpha_v \) is the volumetric thermal expansion coefficient at different temperature. Besides the frequency shift, the change of wavevector \( q \) due to thermal expansion is also considered in this work.

QHA is plausible in most materials, especially at low and moderate temperatures owing to small phonon populations. However, for strongly anharmonic materials, the intrinsic anharmonicity due to the cubic, quartic, and higher-order terms of the interatomic potential can cause strong interaction and energy exchange between normal modes, thereby introducing nontrivial temperature-dependent phonon renormalization, which are not accounted for by QHA. For SnSe, the measurement (Figure 3) shows that the temperature-driven change in phonon dispersion (the second item in eq 4) contributes to over 90% of the total changes in phonon frequencies. This very strong phonon softening effect, not fully
accounted for by thermal expansion, indicates that the quasi-harmonic model may not be appropriate for SnSe. At room temperature, such a strong lattice softening is only observable when a phase change of the material is approached, but usually not for a single material phase.\textsuperscript{44} At high temperature, \(\sim 1500\) K, a recent measurement using inelastic neutron scattering shows consistent phonon shifts in silicon.\textsuperscript{45} Therefore, the measured temperature effect here suggests a re-examination of the ubiquitous theory models\textsuperscript{37} for SnSe at room temperature and even below.

To take account of the phonon softening due to intrinsic anharmonicity, we apply self-consistent phonon theory\textsuperscript{43–45} to calculate the frequency shift, i.e., anharmonicity (AH) model. Essentially, back to the original phonon picture following the idea of using quantized harmonic oscillators to describe atomic lattice vibrations, the AH accounts interactions and energy exchange between the normal quantization modes.\textsuperscript{3} There are two theories to consider the anharmonic effect to phonon frequency shift, perturbation theory, and self-consistent phonon theory. Within perturbation theory,\textsuperscript{46–51} the anharmonic correlation is far smaller than the harmonic frequency, and the frequency shift due to anharmonicity is given by the real part of self-energy as \(\Delta\omega_{\lambda}^{AH} = -\frac{1}{\hbar} Re\Sigma_\lambda\), where \(\Sigma_\lambda\) is the self-energy of phonon mode \(\lambda\). However, when the anharmonic terms is very large, a nonperturbative self-consistent theory\textsuperscript{52,53} should be applied, where the anharmonic phonon frequencies \(\{\omega_\lambda\}\) and harmonic phonon frequencies \(\{\omega_\lambda\}\) can be linked as

\[
\Omega_\lambda^2 = \omega_\lambda^2 + 2\Omega_\lambda I_\lambda
\]

where \(I_\lambda\) is associated with fourth-order IFCs if only considering the first-order self-energy

\[
I_\lambda = -\sum_{\lambda_1} \frac{\hbar \Phi_{\lambda,\lambda_1} - \hbar \lambda_1, -\lambda_1}{4\Omega_\lambda \Omega_{\lambda_1}} 2n_i(\Omega_{\lambda_1}) + 1
\]

where \(\Phi_{\lambda,\lambda_1,\lambda_2,\ldots,\lambda_n}\) is the reciprocal representation of the \(n\)-th IFCs. \(\lambda = (q, j)\) labels a phonon mode with wave vector \(q\) and polarization \(j\), and \(-\lambda = (-q, j)\). \(n_i\) is the Bose–Einstein distribution function for phonon mode \(\lambda_1\). Since the high-order self-energy is usually much smaller than the first-order self-energy, we only consider the first-order self-energy in this Letter.

Note that prior work has observed strong anharmonicity in SnSe.\textsuperscript{12,13,15,16} However, most calculations were using parabolic interatomic potential or QHA. Here, we include high-order anharmonic contributions to calculate the temperature-dependent phonon dispersion of SnSe in the AH model.\textsuperscript{3} Our \textit{ab initio} calculation on temperature-dependent phonon dispersion of SnSe can be divided into three steps. First, density functional theory (DFT) was applied to find an optimized equilibrium structure with force acting on each ion less than \(10^{-4}\) eV/Å. Second, the second and fourth order IFCs were calculated by finite displacement method. We generated irreducible sets of displacement on a supercell using ALAMODE package, calculated the force acting on each ion by DFT, and derived IFCs based that. For the third order IFCs, we consider up to the eighth nearest neighboring atoms for Sn atoms and up to the fifth nearest neighboring atoms for Se atoms. For fourth order IFCs, we consider up to the fourth nearest neighboring atoms for both Sn atoms and Se atoms. The DFT calculations are accomplished using Quantum ESPRESSO.\textsuperscript{3} For all the DFT calculations, we used projector augmented-wave pseudopotential with local density approximation,\textsuperscript{57} and the electronic wave functions were expanded in plane-wave basis. Third, we solved eq 8 using ALAMODE package\textsuperscript{3} to get temperature-dependent dispersion relation of SnSe.

Figure 4 displays our \textit{ab initio} calculation results considering different mechanisms. Figure 4b shows the DFT calculated phonon dispersion in good agreement with inelastic neutron scattering experiment.\textsuperscript{5} The calculation results of the phonon dispersions at different temperatures are shown in Figure 4c–e, with the comparison between HA, QHA, and AH. Figure 4c shows the temperature-dependent phonon dispersions from anharmonic models \((\omega_\lambda^{AH})\). In Figure 4d, we calculate the quasi-harmonic frequency shift from 120 K \((\Delta\omega_\lambda^{QHA})\), based on our measured volumetric thermal expansion coefficient \(\alpha_v\) from 120 to 300 K and first-principles calculated mode-dependent Grüneisen parameter. We take the phonon dispersion from the anharmonic model at 120 K as the starting point for QHA calculation. While a decrease in phonon frequency with increasing temperature is observed in both models, it clearly shows that the AH has a stronger effect than QHA. Combining both QHA and AH effects, i.e., \(\omega_\lambda^{total} = \Delta\omega_\lambda^{QHA} + \omega_\lambda^{AH}\), the phonon dispersion is updated in Figure 4e. Furthermore, in Figure 4f, the longitudinal cross-plane sound velocity \((v)\) of SnSe is extracted and plotted in comparison with our experimental data shown in Figure 3e. In HA, the sound velocity is independent from temperature. In QHA, the sound velocity is changed with temperature only due to thermal expansion, and for SnSe, the temperature dependence of sound velocity within QHA is much weaker than experiment. After including the AH, a much stronger temperature dependence in sound velocity is predicted from 100 to 300 K, consistent with our experimental measurement. In Figure 4f, we also compared the volumetric thermal expansion coefficient \((\alpha_v)\) of SnSe from our experimental measurement with the prediction using the QHA method.\textsuperscript{17} The QHA calculated result does not include the AH and clearly underestimates the thermal expansion, showing a larger deviation from the experimental data with increased temperature. Therefore, this study verifies the failure of QHA for SnSe even below room temperature and confirms the importance of high-order anharmonic effects in understanding its thermal properties.

In summary, we performed nanoscale anisotropic thermal transport on truly dense SnSe single-crystals and measured their intrinsic thermal conductivity to be highly anisotropic. Our temperature-dependent data from 120 to 300 K verifies that the anharmonic phonon scattering dominates transport in high purity single-crystals. Furthermore, we successfully measured the temperature-dependent sound velocity, thermal expansion coefficient, and the Grüneisen parameter at the Brillouin zone center using picosecond laser ultrasonics and XRD. We also performed \textit{ab initio} calculations of phonon band structures using harmonic, quasi-harmonic, and anharmonic models. Our experimental study and modeling based on self-consistent theory show a very strong temperature-dependent intrinsic anharmonicity in SnSe at room temperature or below and indicate that the harmonic and quasi-harmonic models that are widely used in the current phonon calculations fail to accurately predict these important thermophysical properties of SnSe. Our study represents a significant research benchmark for high-performance thermal materials and establishes a
fundamental insight into improving the ab initio calculations for modern phonon transport theory.

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S1. Synthesis of SnSe crystals

High quality single-crystal SnSe was synthesized through a repeated multi-step chemical vapor transport process. Details of the chemical vapor transport setups and synthesize process to grow high-quality single crystals can be found in our recent publications [S1,S2,S5]. Stoichiometric ratio of high-purity Tin (Sn) and selenium (Se) (99.999% purity, from Alfa Aesar) were placed into a quartz tube. This tube was evacuated and flame sealed under high vacuum (10^{-5} torr). A customized multi-zone reactor was set up with reaction temperatures of 1223 K and 1173 K for the hot and cold zones, respectively. Temperature changes were controlled with a very slow rate for over 12 hours. The reactor was held at the reaction temperatures for over 96 hours and then cooled down to room temperature. The initial growth led to crystal ingots. The as-grown crystals were pulverized and returned to the quartz tube, which was evacuated, flame sealed, and loaded into the reactor. This growth procedure was repeated until high quality single crystal SnSe with the close-to-theory maximum density were obtained.

S2. Nanoscale optic-thermal measurement

Measurement of the intrinsic thermal conductivity of single crystal SnSe was performed using the time-domain thermoreflectance (TDTR), which has become a standard technique to measure the thermal conductivity of solids [S1-S8]. TDTR is well suited to study the intrinsic thermal conductivity of SnSe samples because no physical contact with the sample is required and the measurements can be performed over a region as small as ~ 1 micrometer in size. In the TDTR setup (Fig. S1a), a laser beam pump consisting of a train of short pulses (~ 50 femtoseconds) created an instantaneous temperature rise at the sample surface. The transient temperature decay, from the heat impulse was monitored with another probe pulse that was delayed in time with respect to the pump using a mechanical delay stage (with sub-picosecond resolution). The SnSe full transient decay curve was obtained by varying the time delay, and was subsequently fitted to a thermal model to deduce the thermal conductivity (Fig. S1c). In our setup
(Fig. S1b), a Ti:Sapphire oscillator generated 800 nm wavelength, 50~100 femto-second pulse width, and 80MHz repetition rate laser pulses. The beam was split into pump and probe beams with a large power ratio. The pump beam frequency was sinusoidally modulated with an electric optic modulator (EOM) up to 20MHz, and the fundamental frequency of the pump beam was doubled by bismuth triborate crystal (BIBO). The probe beam was delayed with a mechanical delay stage from 0 to 6000 ps. Both pump and probe beams were focused on the sample surface with an objective lens and the reflected signal of the probe beam was detected by an Si photodiode (Thorlabs PDA36A) connected to a lock-in amplifier for exclusive signal detection of the modulation frequency of the pump.

Mathematically, the detected signal can be expressed as:

\[
Z(f_0) = \sum_{N=-\infty}^{\infty} H(f_0 + Nf_s) e^{iNf_s \tau}
\]

where \(H(f)\) is the response frequency, \(f_s\) the pulse repetition frequency, and \(\tau\) the time delay between the pump and probe beams. The response frequency function can be solved with the thermal diffusion model. For our measurements on SnSe, a one-dimensional heat conduction model was solved in the frequency domain. Additional details can be found in the literature [S4]. Amplitude and phase changes indicate a temperature change of the metal film and are functions of the time delay. The thermal conductivity of SnSe samples was obtained by fitting the experimental data to this theoretical model. Fig. S1(c) shows a typical TDTR phase change result along the c-axis of SnSe with its best fit to the thermal diffusion model. By intentionally changing the thermal conductivity values by ±10 %, two other phase change curves were plotted (dashed black lines). The distribution of all the experimental data within the narrow range between the two dashed curves demonstrates the high accuracy of our thermal conductivity measurements. A detailed description of our nanoscale optic-thermal measurement technique and anisotropic thermal measurement can be found in our recent papers [S1-S5, S8].
Figure S1. Time-domain thermoreflectance (TDTR) setup and a typical measurement on SnSe. (a) Schematic of the TDTR setup; (b) a photograph of the TDTR setup; (c) a typical TDTR measurement result (red circle, for the c-axis of SnSe), fitted to the thermal transport model (solid blue line). Plots of the calculated curves (black dashed lines), using the thermal conductivity changed by ±10% of the best values, support a measurement uncertainty within 10%.

S3. Specific heat of SnSe

The specific heat of SnSe was measured across the temperature range from 120 – 300 K, using the differential scanning calorimetry (PerkinElmer DSC 8000), the two furnaces system and the StepScan mode of which ensured the measurement accuracy. The measurement parameters were set as 10 K/min.
for the heating rate, 10 K for the temperature interval, 1 min for thermalization time and 20 mL/min for the nitrogen purge flow rate. The temperature-dependent specific heat of SnSe was plotted by averaging ten repetitive measurement results (Fig. S3) [S10].

![Graph showing temperature-dependent specific heat of single-crystal SnSe.](image)

**Figure S3.** The temperature dependent specific heat of single-crystal SnSe.

### S4. Derivation of the Grüneisen parameter at the Brillouin zone center

The phonon frequency \( \omega_i \) depends on the material volume \( V \) and the temperature \( T \) \( \omega_i (V, T) \). The mode dependent Grüneisen parameter \( \gamma_i \) and phonon group velocity \( v_{gi} \) are defined by [S9]:

\[
\gamma_i = -\frac{V}{\omega_i} \frac{\partial \omega_i}{\partial V} \quad (S2)
\]

\[
v_{gi} = \frac{\partial \omega_i}{
\partial q_i} \quad (S3)
\]
where $q_i$ is the reciprocal vector along a specific crystal direction defined as $q_i = \frac{2\pi i}{N \cdot a_i}$ ($a_i$ is the lattice constant).

Near the Brillouin zone center, i.e., for small frequency phonons, $v_{gi}$ is independent of the frequency and is equivalent to the sound velocity ($v_g$),

$$v_{gi} = v_g = \frac{\partial \omega_i}{\partial q_i} = \frac{\omega_i}{q_i} \quad (S4)$$

Under constant pressure,

$$\frac{\partial \omega_i}{\partial T} = \frac{\partial (v_g^2 q_i^2)}{\partial T} = v_g \frac{\partial (v_g)}{\partial T} + q_i \frac{\partial (v_g^2)}{\partial T} = v_g^* q_i^* \frac{\partial (v_g)}{\partial T} + q_i \frac{\partial (v_g^2)}{\partial T} = -v_g^* \frac{\partial (v_g)}{\partial T} = \frac{\omega_i}{v_g} \frac{\partial (v_g)}{\partial T} = -\frac{\omega_i}{v_g} \frac{\partial (v_g)}{\partial T} \quad (S5)$$

where $\alpha_i$ is the linear thermal expansion coefficient along the specific crystal orientation.

Therefore,

$$\gamma_i = -\frac{V}{\omega_i} \frac{\partial \omega_i}{\partial V} = -\frac{\partial \omega_i}{\omega_i \cdot \partial T} = \frac{\partial V}{V \cdot \partial T} = -\frac{\alpha_i}{\omega_i \cdot \partial T} / \alpha_V \quad (S7)$$

where $\alpha_V$ is the volumetric thermal expansion coefficient. From Equation (S6) and (S7), we have,

$$\gamma_i = \frac{\alpha_i}{\alpha_V} \frac{1}{v_g^* v_g} \frac{\partial (v_g^2)}{\partial T} \quad (S8)$$

S5. *Ab initio* calculations

We performed *ab initio* calculations of temperature dependent phonon dispersions of SnSe following three steps. First, density functional theory (DFT) was applied to find an optimized equilibrium structure with force acting on each ion less than $10^{-4}$ eV/Å. Second, the second and fourth order IFCs were calculated by finite displacement method. We built a 144 atoms supercell, generated irreducible sets of displacement using ALAMODE package, calculated the force acting on each ion by DFT, and derived IFCs based that. For the 3rd order IFCs, we consider up to 8th nearest neighboring atoms for Sn atoms and up to 5th nearest neighboring atoms for Se atoms. For 4th order IFCs, we consider up to 4th nearest
neighboring atoms for both Sn atoms and Se atoms. The DFT calculations are accomplished using
Quantum ESPRESSO[S11, S12]. For all the DFT calculations, we used projector augmented-wave
pseudopotential with local density approximation [S13], and the electronic wave functions were expanded
in plane-wave basis with kinetic energy cutoff at 544 eV. We used $4 \times 6 \times 6$ Monkhorst-Pack k-points
meshes to sample the Brillouin zone for structure optimization, and $2 \times 2 \times 2$ Monkhorst-Pack k-points
meshes for IFCs calculation. Third, we solved Eq. (8) using ALAMODE package[S14, S15] to get
temperature dependent dispersion relation of SnSe.

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S13. We used the pseudopotentials Sn.pz-dn-kjpaw_psl.0.2.UPF and Se.pz-n-kjpaw_psl.0.2.UPF from http://www.quantum-espresso.org.
